The role of magnetoplasmons in Casimir force calculations

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Abstract

In this paper we review the role of magneto plasmon polaritons in the Casimir force calculations. By applying an external constant magnetic field a strong optical anisotropy is induced on two parallel slabs reducing the reflectivity and thus the Casimir force. As the external magnetic field increases, the Casimir force decreases. Thus, with an an external magnetic field the Casimir force can be controlled.The calculations are done in the Voigt configuration where the magnetic field is parallel to the slabs. In this configuration the reflection coefficients for TE and TM modes do not show mode conversion.

Keywords: Casimir, magnetoplasmons, Voigt

I. INTRODUCTION

The optics of surfaces plays an important role in the calculation of the Lifshitz-Casimir formula [1]. Indeed the Lifshitz formula can be obtained from the sum of surface-polariton modes between two metallic slabs [2, 3, 4, 5, 6]. In metallic and semiconducting surfaces the effect of an externally applied magnetic field $\mathbf{B_0}$ leds to the excitation of magnetoplasmon modes. This external magnetic field changes significantly the behavior of the plasma modes and induces an optical anisotropy that is magnetic field dependent [7]. This has the effect of reducing the Casimir force significantly. This reduction on the force has been applied to the problem of pull-in dynamics in micro and nano electrodynamical systems (mems and nems), and as has been shown to increase the detachment length in cantilever mems and nems [8].

To illustrate this point, consider the Drude model in the presence of the external magnetic field. The equation of motion for the electrons in the material is

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{m}{\tau}\mathbf{v} \tag{1}$$

where m is the effective mass of the electron, q the charge and τ is the relaxation time. Assuming an harmonic electric field $e^{-i\omega t}$ the current $\mathbf{j} = nq\mathbf{v}$ can be found and thus the conductivity can be calculated [9]

$$\sigma_{ij}(\omega, \mathbf{B}_0) = \frac{nq^2}{\tau^* m}\frac{\delta_{ij} + \omega_c\tau^* e_{ijk}(B_k/B_0) + (wc\tau^*)^2(B_iB_j/B_0^2)}{1 + (\omega_c\tau^*)^2}, \tag{2}$$

where $\tau^* = \tau/(1 - i\omega\tau)$, $w_c = q|\mathbf{B}_0|/mc$ is the cyclotron frequency, and e_{ijk} is the Levi-Civita symbol. The dielectric tensor is obtained from

$$\epsilon_{ij}(\omega, \mathbf{B}_0) = \delta_{ij} + \frac{4\pi i}{\omega}\sigma_{ij}. \tag{3}$$

Clearly if $\mathbf{B}_0 = 0$ we recover the results for the isotropic case.

II. VOIGT AND FARADAY CONFIGURATIONS

For an arbitrary direction of the magnetic filed, the calculation of the dispersion relation of the surface magneto plasmons and of the optical reflectivity is difficult. To simplify the problem, specific directions of the magnetic field have to be chosen [10]. In the so called

Faraday configuration, the magnetic field is perpendicular to the slab. In this case, there is mode conversion upon reflection from the slab. This is, if a TE wave is incident, the reflected wave will consists of a TE and TM modes, similar for an incident TM mode.

The second configuration, that will be used in this paper, is the *Voigt* geometry where the magnetic field is parallel to the slabs. In this case there is no mode conversion upon reflection. Consider a slab parallel to the $x-z$ plane. In the Voigt geometry the external magnetic field points along the z axis. In this case, the components of the dielectric tensor are given by [10, 11]

$$\begin{aligned}
\epsilon_{xx} &= \epsilon_L\left[1-\frac{\omega_p^2}{\omega^2}\right], \\
\epsilon_{yy} &= \epsilon_L\left[1-\frac{\omega_p^2}{\omega^2-\omega_c^2}\right], \\
\epsilon_{yz} &= \epsilon_L\left[\frac{i\omega_c\omega_p^2}{\omega(\omega^2-\omega_c^2)}\right],
\end{aligned} \tag{4}$$

and $\epsilon_{zz}=\epsilon_{yy}$ and $\epsilon_{zy}=-\epsilon_{yz}$. The other components are equal to zero. In these equations ϵ_L is the background dielectric function, ω_p the plasma frequency, In the absence of the magnetic field, $\omega_c=0$ and the plates become isotropic. For simplicity we have not included the Drude damping parameter. In the rest of the paper we will use the dimensionless variable $\Omega_c=\omega_c/\omega_p$, that gives the relative importance of the external magnetic field. In Figure (1) we have plotted the dielectric function components as given by Eq. (4) after rotation of the frequency to the complex plane $\omega \rightarrow i\zeta$ for a value of $\Omega_c=0.2$, showing the strong anisotropy of the system.

In the material slab, the dispersion relation is obtained from the wave equation

$$\nabla\times\nabla\times\mathbf{E}-q_0^2\tilde{\epsilon}\cdot\mathbf{E}=0, \tag{5}$$

where $\tilde{\epsilon}$ is the dielectric tensor and $q_0^2=\omega^2/c^2=q_x^2+q_y^2+q_z^2$. Upon replacement of its components (Eq.(4)) into Eq.(5), the nontrivial solution of the resulting equation is obtained if

$$-q_y^2=\beta^2=q_z^2-q_0^2(\epsilon_{zz}+\frac{\epsilon_{yz}^2}{\epsilon_{ss}}). \tag{6}$$

Outside the slab we assume there is vacuum and we have the usual dispersion

$$-q_y^2=\alpha^2=q_z^2-q_0^2. \tag{7}$$

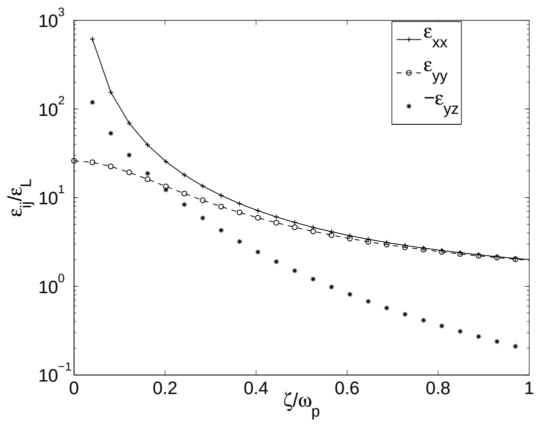

FIG. 1: Components of the dielectric function in the Voigt configuration (Eq. (4)) for $\Omega_c = 0.2$. The dielectric function is evaluated in the rotated frequency axis.

The reflection coefficient can now be calculated taking into account that outside the slab, for a TM polarized wave the field is of the form

$$B_x(r,t) = A_{\pm} e^{\pm \alpha y} e^{i(q_z z - \omega t)}, \tag{8}$$

and within the slab

$$B_x(y) = C_{\pm} e^{\pm \beta y} e^{i(q_z z - \omega t)}, \tag{9}$$

where β and α are given by Eqs.(6) and (7) respectively. From the corresponding electric fields and by applying the boundary conditions, the reflection coefficients can be found. The detailed procedure for the Voigt configuration can be found in Ref.([10]).

III. REDUCTION OF THE CASIMIR FORCE WITH AN EXTERNAL MAGNETIC FIELD

To study the effect of the external magnetic field on the Casimir force we use Lifhitz formula

$$F = \frac{k_B T}{8\pi L^2} \sum_{n=0}^{\infty} \int_{\zeta_n}^{\infty} q_y dq_y \frac{1}{r_s^{-2} e^{2q_y L} - 1} + (r_s \rightarrow r_p), \tag{10}$$

where $\zeta_n = 2\pi k_B T n/\hbar$ is the Matsubara frequencies and r_{sp} the reflectivities for p or s polarized modes (TM and TE, respectively). This expression for the Casimir force can

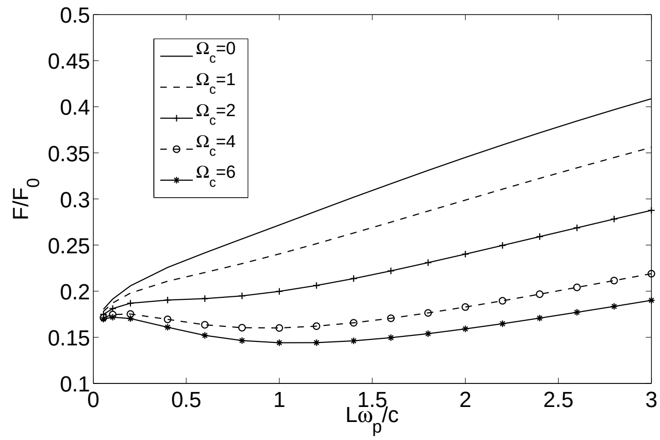

FIG. 2: The Casimir force normalized to the ideal case as a function of separation. The different curves correspond to different values of Ω_c. As the external magnetic field increases, Ω_c also increases and the Casimir force decreases. The separation between the plates is in terms of c/ω_p.

be used only for the Voigt configuration since there is no mode conversion. The Faraday configuration, that will be presented elsewhere, requires a more general expression for the Casimir force, as the one discussed by Bruno [12]. The reflectivities r_p and r_s are replaced in Eq. (10) by a reflectivity matrix whose components are r_{pp}, r_{ps}, r_{ss}, r_{sp}, where the first subindex represents the polarization of the incident wave and the second sub-index the polarization of the reflected wave.

In Figure (2) we plot the Casimir force normalized to the ideal case $F_0 = -\hbar c\pi^2/240L^4$, for several values of the reduced frequency Ω_c. All frequencies are normalized to the plasma frequency and the distances are normalized to the plasma wavelength c/ω_p. The value for the background dielectric function $\epsilon_L = 15.4$ is for $InSb$ as reported by Palik [13]. In general, $III-V$ semiconductors (e.g. $GaAs, GaN, InAs$) can be used, since they exhibit a strong magnetoplasmon response. The important feature of Fig. (2), is that as the magnetic field increases the Casimir force decreases. For high magnetic fields, there is a drop in F/F_0 as a function of separation, this drop is more significant with increasing magnetic field.

IV. CONCLUSIONS

In this paper we have reviewed briefly the principles of magnetoplasmons in semiconductors and its effect on the calculations of the Casimir force between parallel slabs. In

particular we consider the Voigt configuration, where the magnetic field is parallel to the surface of the slabs. This external magnetic field induces a strong optical anisotropy that reduces the reflectivity. This has the effect of reducing the Casimir force as the external magnetic field increases. In a future work, the effect of the external magnetic field on the Casimir torque will be considered.

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